

Code Generation for Data Processing

Lecture 3: Intermediate Representations

Alexis Engelke

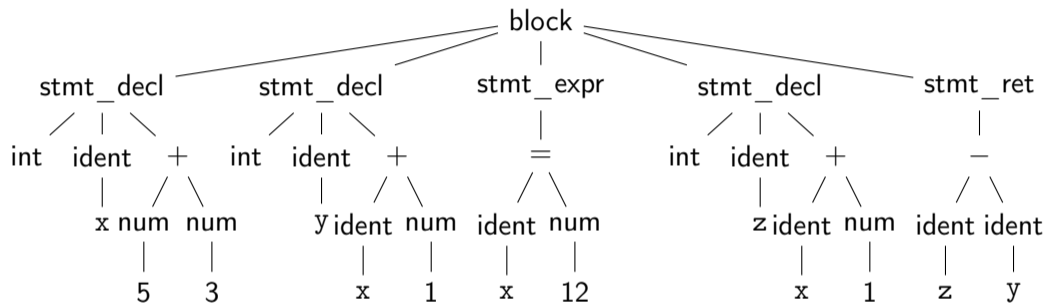
Chair of Data Science and Engineering (I25)
School of Computation, Information, and Technology
Technical University of Munich

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Intermediate Representations: Motivation

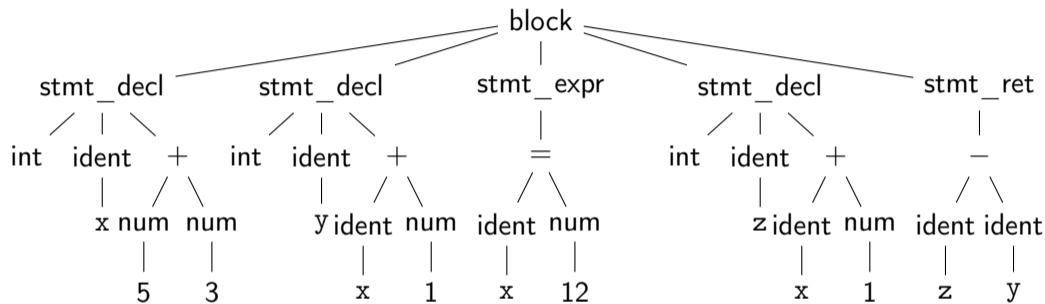
- ▶ So far: program parsed into AST
- + Great for language-related checks
- + Easy to correlate with original source code (e.g., errors)
- Hard for analyses/optimizations due to high complexity
 - ▶ variable names, control flow constructs, etc.
 - ▶ Data and control flow implicit
- Highly language-specific

Intermediate Representations: Motivation



Question: how to optimize? Is `x+1` redundant?

Intermediate Representations: Motivation



Question: how to optimize? Is $x+1$ redundant? \rightsquigarrow hard to tell ☹️

Intermediate Representations: Motivation

```
x1 ← 5 + 3
y1 ← x1 + 1
x2 ← 12
z1 ← x2 + 1
tmp1 ← z1 - y1
return tmp1
```

Question: how to optimize? Is $x+1$ redundant?

Intermediate Representations: Motivation

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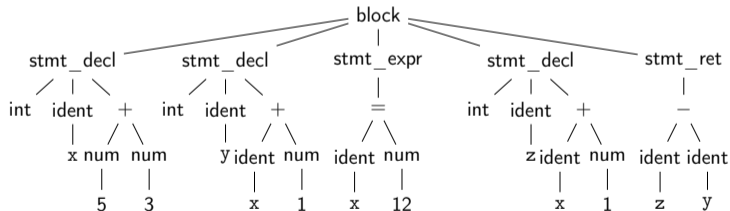
Question: how to optimize? Is $x+1$ redundant? \rightsquigarrow No! 😊

Intermediate Representations

- ▶ Definitive program representation inside compiler
 - ▶ During compilation, only the (current) IR is considered
- ▶ Goal: simplify analyses/transformations
 - ▶ *Technically*, single-step compilation is possible for, e.g., C ... but optimizations are hard without proper IRs
- ▶ Compilers *design* IRs to support frequent operations
 - ▶ IR design can vary strongly between compilers
- ▶ Typically based on **graphs** or **linear instructions** (or both)

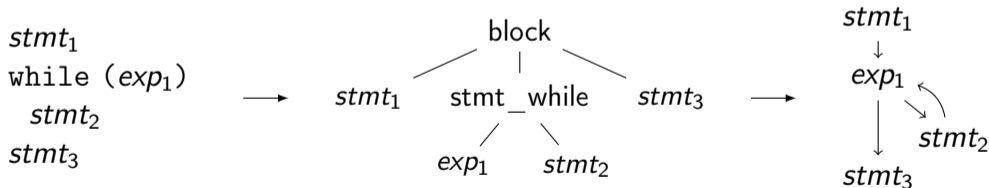
Graph IRs: Abstract Syntax Tree (AST)

- ▶ Code representation close to the source
- ▶ Representation of types, constants, etc. might differ
- ▶ Storage might be problematic for large inputs



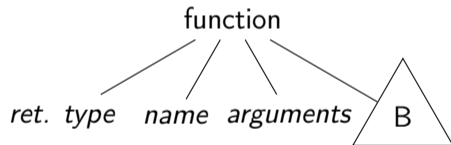
Graph IRs: Control Flow Graph (CFG)

- ▶ Motivation: model control flow between different code sections
- ▶ Graph nodes represent **basic blocks**
 - ▶ Basic block: sequence of branch-free code (modulo exceptions)
 - ▶ Typically represented using a linear IR



Build CFG from AST – Function

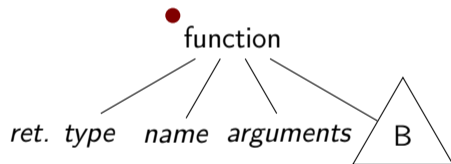
- ▶ Idea: Keep track of current insert block while walking through AST



Build CFG from AST – Function

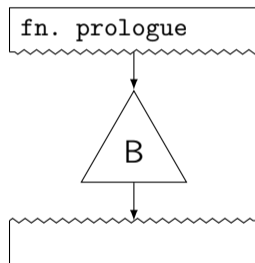
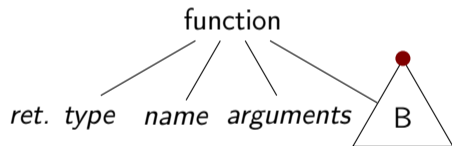
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fn. prologue



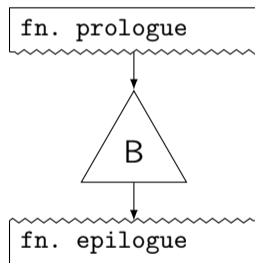
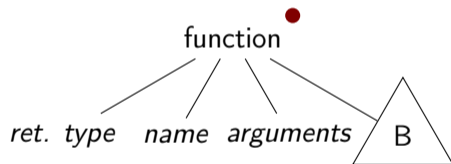
Build CFG from AST – Function

- ▶ Idea: Keep track of current insert block while walking through AST



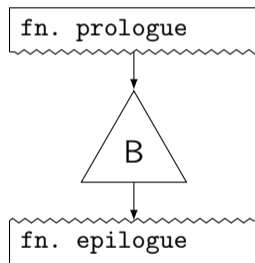
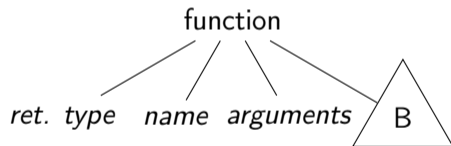
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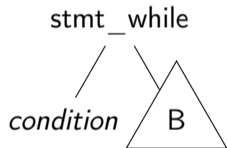
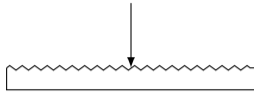


Build CFG from AST – Function

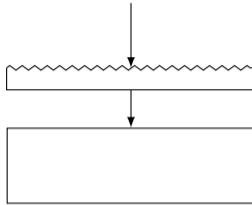
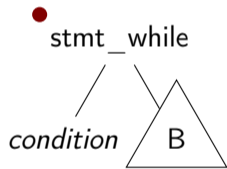
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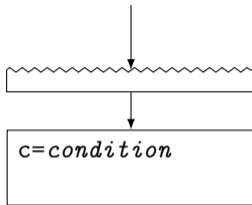
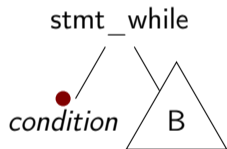
Build CFG from AST – While Loop



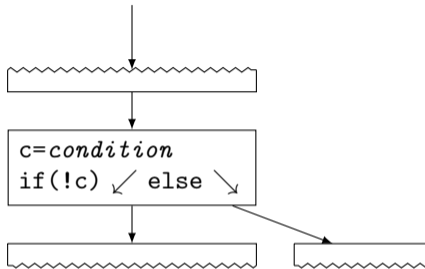
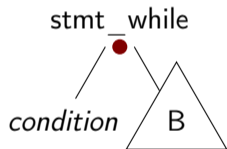
Build CFG from AST – While Loop



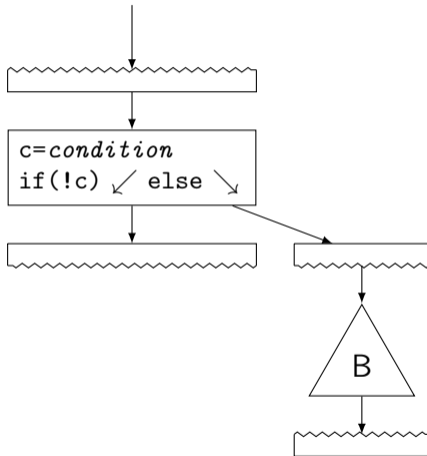
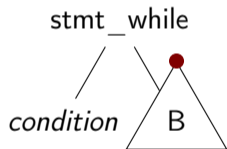
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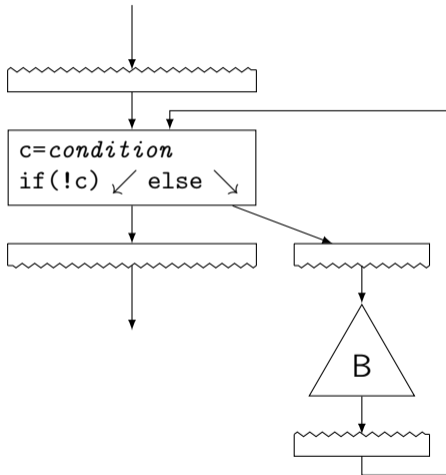
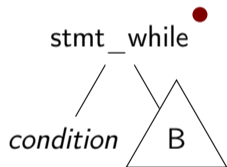
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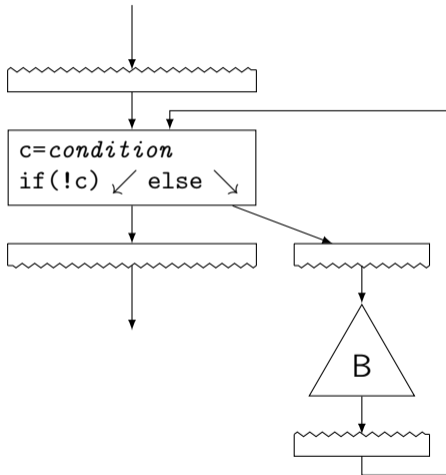
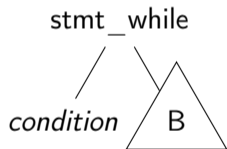
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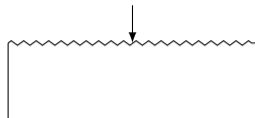
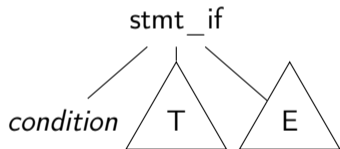
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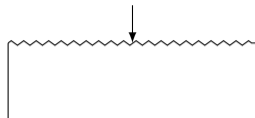
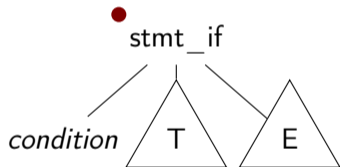
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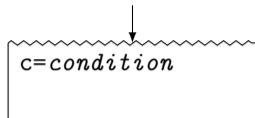
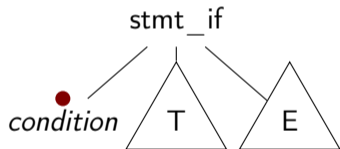
Build CFG from AST – If Condition



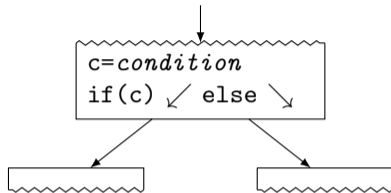
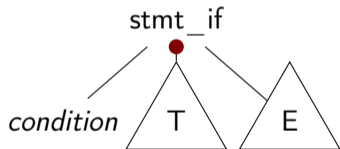
Build CFG from AST – If Condition



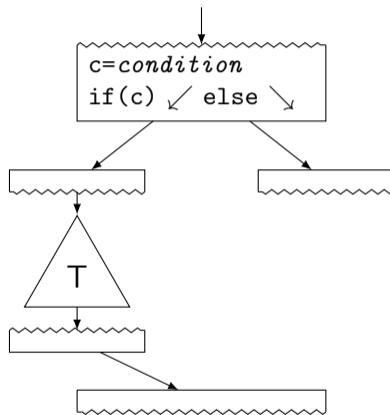
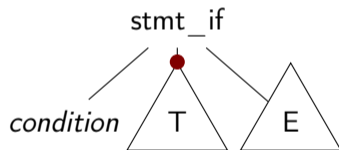
Build CFG from AST – If Condition



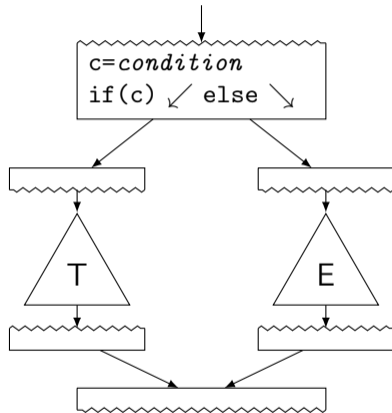
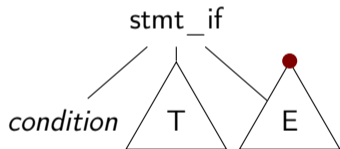
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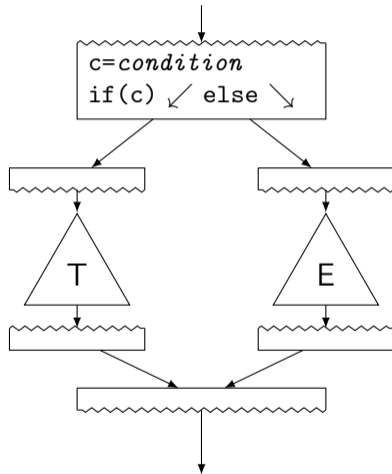
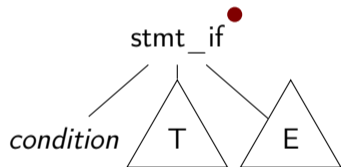
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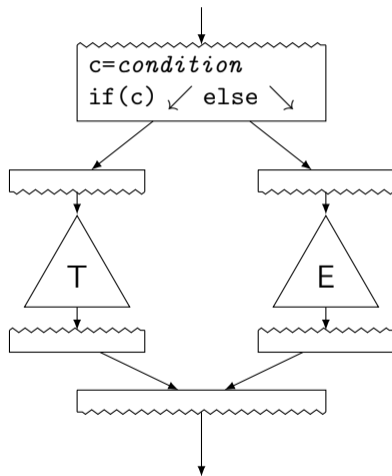
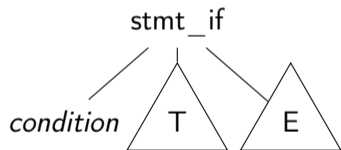
Build CFG from AST – If Condition



Build CFG from AST – If Condition



Build CFG from AST – If Condition



Build CFG from AST: Switch

Build CFG from AST: Switch

Linear search

```
t ← exp
if t == 3: goto B3
if t == 4: goto B4
if t == 7: goto B7
if t == 9: goto B9
goto BD
```

Binary search

```
t ← exp
if t == 7: goto B7
elif t > 7:
    if t == 9: goto B9
else:
    if t == 3: goto B3
    if t == 4: goto B4
goto BD
```

Jump table

```
t ← exp
if 0 ≤ t < 10:
    goto table[t]
goto BD

table = {
    BD, BD, BD, B3,
    B4, BD, ... }
```

Build CFG from AST: Switch

Linear search

```
t ← exp
if t == 3: goto B3
if t == 4: goto B4
if t == 7: goto B7
if t == 9: goto B9
goto BD
```

- + Trivial
- Slow, lot of code

Binary search

```
t ← exp
if t == 7: goto B7
elif t > 7:
    if t == 9: goto B9
else:
    if t == 3: goto B3
    if t == 4: goto B4
goto BD
```

- + Good: sparse values
- Even more code

Jump table

```
t ← exp
if 0 ≤ t < 10:
    goto table[t]
goto BD

table = {
    BD, BD, BD, B3,
    B4, BD, ... }
```

- + Fastest
- Table can be large, needs ind. jump

Build CFG from AST: Break, Continue, Goto

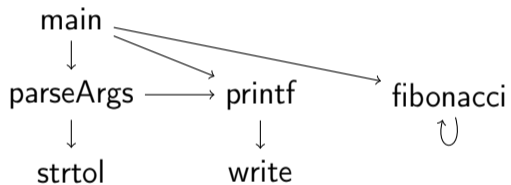
- ▶ break/continue: trivial
 - ▶ Keep track of target block, insert branch
- ▶ goto: also trivial
 - ▶ Split block at target label, if needed
 - ▶ But: may lead to irreducible control flow graph

CFG: Formal Definition

- ▶ **Flow graph:** $G = (N, E, s)$ with a digraph (N, E) and entry $s \in N$
 - ▶ Each node is a basic block, s is the entry block
 - ▶ $(n_1, n_2) \in E$ iff n_2 might be executed immediately after n_1
 - ▶ All $n \in N$ shall be reachable from s (unreachable nodes can be discarded)
 - ▶ Nodes without successors are end points

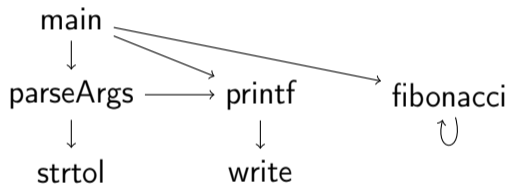
Graph IRs: Call Graph

- ▶ Graph showing (possible) call relations between functions



Graph IRs: Call Graph

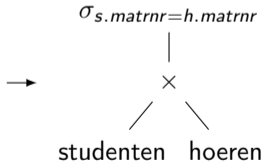
- ▶ Graph showing (possible) call relations between functions
- ▶ Useful for interprocedural optimizations
 - ▶ Function ordering
 - ▶ Stack depth estimation
 - ▶ ...



Graph IRs: Relational Algebra

- ▶ Higher-level representation of query plans
 - ▶ Explicit data flow

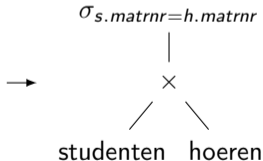
```
SELECT s.name, h.vorlnr  
FROM studenten s, hoeren h  
WHERE s.matrnr = h.matrnr
```



Graph IRs: Relational Algebra

- ▶ Higher-level representation of query plans
 - ▶ Explicit data flow
- ▶ Allow for optimization and selection actual implementations
 - ▶ Elimination of common sub-trees
 - ▶ Joins: ordering, implementation, etc.

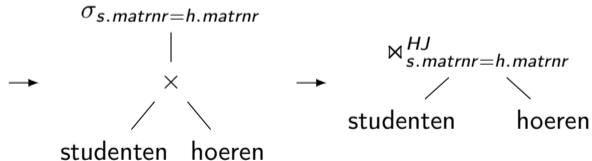
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Graph IRs: Relational Algebra

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```
SELECT s.name, h.vorlnr  
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Linear IRs: Stack Machines

- ▶ Operands stored on a stack
- ▶ Operations pop arguments from top and push result
- ▶ Typically accompanied with variable storage
- ▶ Generating IR from AST: trivial
- ▶ Often used for bytecode, e.g. Java, Python

+

-

```
push 5
push 3
add
pop x
push x
push 1
add
pop y
push 12
pop x
push x
push 1
add
pop z
```


Linear IRs: Stack Machines

- ▶ Operands stored on a stack
 - ▶ Operations pop arguments from top and push result
 - ▶ Typically accompanied with variable storage
 - ▶ Generating IR from AST: trivial
 - ▶ Often used for bytecode, e.g. Java, Python
- + Compact code, easy to generate and implement
- Performance, hard to analyze

```
push 5
push 3
add
pop x
push x
push 1
add
pop y
push 12
pop x
push x
push 1
add
pop z
```

Linear IRs: Register Machines

- ▶ Operands stored in registers
- ▶ Operations read and write registers
- ▶ Typically: infinite number of registers
- ▶ Typically: three-address form
 - ▶ $dst = src1 \ op \ src2$
- ▶ Generating IR from AST: trivial
- ▶ E.g., GIMPLE, eBPF, Assembly

```
x    ←  5  +  3
y    ←  x  +  1
x    ←  12
z    ←  x  +  1
tmp1 ←  z  -  y
return    tmp1
```

Example: High GIMPLE

```
int fac (int n)
gimple_bind < // <-- still has lexical scopes
  int D.1950;
  int res;

  gimple_assign <integer_cst, res, 1, NULL, NULL>
  gimple_goto <<D.1947>>
  gimple_label <<D.1948>>
  gimple_assign <mult_expr, _1, n, n, NULL>
  gimple_assign <mult_expr, res, res, _1, NULL>
  gimple_assign <plus_expr, n, n, -1, NULL>
  gimple_label <<D.1947>>
  gimple_cond <ne_expr, n, 0, <D.1948>, <D.1946>>
  gimple_label <<D.1946>>
  gimple_assign <var_decl, D.1950, res, NULL, NULL>
  gimple_return <D.1950>
>

int foo(int n) {
  int res = 1;
  while (n) {
    res *= n * n;
    n -= 1;
  }
  return res;
}
```

\$ gcc -fdump-tree-gimple-raw -c foo.c

Example: Low GIMPLE

```
int fac (int n)
{
    int res;
    int D.1950;

int foo(int n) {
    int res = 1;
    while (n) {
        res *= n * n;
        n -= 1;
    }
    return res;
}

    gimple_assign <integer_cst, res, 1, NULL, NULL>
    gimple_goto <<D.1947>>
    gimple_label <<D.1948>>
    gimple_assign <mult_expr, _1, n, n, NULL>
    gimple_assign <mult_expr, res, res, _1, NULL>
    gimple_assign <plus_expr, n, n, -1, NULL>
    gimple_label <<D.1947>>
    gimple_cond <ne_expr, n, 0, <D.1948>, <D.1946>>
    gimple_label <<D.1946>>
    gimple_assign <var_decl, D.1950, res, NULL, NULL>
    gimple_goto <<D.1951>>
    gimple_label <<D.1951>>
    gimple_return <D.1950>
}
```

```
$ gcc -fdump-tree-lower-raw -c foo.c
```

Example: Low GIMPLE with CFG

```
int foo(int n) {
  int res = 1;
  while (n) {
    res *= n * n;
    n -= 1;
  }
  return res;
}

int fac (int n) {
  int res;
  int D.1950;
  <bb 2> :
  gimple_assign <integer_cst, res, 1, NULL, NULL>
  goto <bb 4>; [INV]
  <bb 3> :
  gimple_assign <mult_expr, _1, n, n, NULL>
  gimple_assign <mult_expr, res, res, _1, NULL>
  gimple_assign <plus_expr, n, n, -1, NULL>
  <bb 4> :
  gimple_cond <ne_expr, n, 0, NULL, NULL>
  goto <bb 3>; [INV]
  else
  goto <bb 5>; [INV]
  <bb 5> :
  gimple_assign <var_decl, D.1950, res, NULL, NULL>
  <bb 6> :
  gimple_label <<L3>>
  gimple_return <D.1950>
}
```

```
$ gcc -fdump-tree-cfg-raw -c foo.c
```

Linear IRs: Register Machines

- ▶ Problem: no clear def–use information
 - ▶ Is $x + 1$ the same?
 - ▶ Hard to track actual values!
- ▶ **How to optimize?**

```
x    ←  5  +  3
y    ←  x  +  1
x    ←  12
z    ←  x  +  1
tmp1 ←  z  -  y
return    tmp1
```

Linear IRs: Register Machines

- ▶ Problem: no clear def–use information

- ▶ Is $x + 1$ the same?
- ▶ Hard to track actual values!

- ▶ **How to optimize?**

⇒ Disallow mutations of variables

```
x    ←  5  +  3
y    ←  x  +  1
x    ←  12
z    ←  x  +  1
tmp1 ←  z  -  y
return    tmp1
```

Single Static Assignment: Introduction

- ▶ Idea: disallow mutations of variables, value set in declaration
- ▶ Instead: create new variable for updated value
- ▶ SSA form: every computed value has a unique definition
 - ▶ Equivalent formulation: each name describes result of one operation

x	\leftarrow	5	$+$	3		x	\leftarrow	5	$+$	3
y	\leftarrow	x	$+$	1		y	\leftarrow	x	$+$	1
x	\leftarrow	12			\longrightarrow	x'	\leftarrow	12		
z	\leftarrow	x	$+$	1		z	\leftarrow	x'	$+$	1
tmp_1	\leftarrow	z	$-$	y		tmp_1	\leftarrow	z	$-$	y
return		tmp_1				return		tmp_1		

Single Static Assignment: Introduction

- ▶ Idea: disallow mutations of variables, value set in declaration
- ▶ Instead: create new variable for updated value
- ▶ SSA form: every computed value has a unique definition
 - ▶ Equivalent formulation: each name describes result of one operation

x	\leftarrow	5	+	3		v_1	\leftarrow	5	+	3
y	\leftarrow	x	+	1		v_2	\leftarrow	v_1	+	1
x	\leftarrow	12			\longrightarrow	v_3	\leftarrow	12		
z	\leftarrow	x	+	1		v_4	\leftarrow	v_3	+	1
tmp_1	\leftarrow	z	-	y		v_5	\leftarrow	v_4	-	v_2
return		tmp_1				return		v_5		

Single Static Assignment: Control Flow

- ▶ How to handle diverging values in control flow?

```
entry :  $x \leftarrow \dots$   
        if ( $x > 2$ ) goto cont    →  
then :  $x \leftarrow x * 2$   
cont : return  $x$ 
```

Single Static Assignment: Control Flow

- ▶ How to handle diverging values in control flow?

```
entry : x ← ...  
       if (x > 2) goto cont  
then : x ← x * 2  
cont : return x
```

→

```
entry : v1 ← ...  
       if (v1 > 2) goto cont  
then : v2 ← v1 * 2  
cont : return ???
```

Single Static Assignment: Control Flow

- ▶ How to handle diverging values in control flow?
- ▶ Solution: Φ -nodes to merge values depending on predecessor
 - ▶ Value depends on edge used to enter the block
 - ▶ All Φ -nodes of a block execute concurrently (ordering irrelevant)

```
entry : x ← ...  
       if (x > 2) goto cont  
then : x ← x * 2  
cont : return x
```

→

```
entry : v1 ← ...  
       if (v1 > 2) goto cont  
then : v2 ← v1 * 2  
cont : v3 ←  $\Phi$ (entry : v1, then : v2)  
       return v3
```

Example: GIMPLE in SSA form

```
int foo(int n) {  
  int res = 1;  
  while (n) {  
    res *= n * n;  
    n -= 1;  
  }  
  return res;  
}
```

```
int fac (int n) { int res, D.1950, _1, _6;  
  <bb 2> :  
  gimple_assign <integer_cst, res_4, 1, NULL, NULL>  
  goto <bb 4>; [INV]  
  <bb 3> :  
  gimple_assign <mult_expr, _1, n_2, n_2, NULL>  
  gimple_assign <mult_expr, res_8, res_3, _1, NULL>  
  gimple_assign <plus_expr, n_9, n_2, -1, NULL>  
  <bb 4> :  
  # gimple_phi <n_2, n_5(D)(2), n_9(3)>  
  # gimple_phi <res_3, res_4(2), res_8(3)>  
  gimple_cond <ne_expr, n_2, 0, NULL, NULL>  
  goto <bb 3>; [INV]  
  else  
  goto <bb 5>; [INV]  
  <bb 5> :  
  gimple_assign <ssa_name, _6, res_3, NULL, NULL>  
  <bb 6> :  
  gimple_label <<L3>>  
  gimple_return <_6>  
}
```

```
$ gcc -fdump-tree-ssa-raw -c foo.c
```

SSA Construction – Local Value Numbering

- ▶ Simple case: inside block – keep mapping of variable to value

Code	SSA IR	Variable Mapping
$x \leftarrow 5 + 3$		
$y \leftarrow x + 1$		
$x \leftarrow 12$		
$z \leftarrow x + 1$		
$tmp_1 \leftarrow z - y$		
return	tmp_1	

SSA Construction – Local Value Numbering

- ▶ Simple case: inside block – keep mapping of variable to value

Code	SSA IR	Variable Mapping
$\rightarrow x \leftarrow 5 + 3$	$v_1 \leftarrow \text{add } 5, 3$	$x \rightarrow v_1$
$y \leftarrow x + 1$		
$x \leftarrow 12$		
$z \leftarrow x + 1$		
$tmp_1 \leftarrow z - y$		
return tmp_1		

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$x \leftarrow 5 + 3$	$v_1 \leftarrow \text{add } 5, 3$	$x \rightarrow v_1$
$\rightarrow y \leftarrow x + 1$	$v_2 \leftarrow \text{add } v_1, 1$	$y \rightarrow v_2$
$x \leftarrow 12$		
$z \leftarrow x + 1$		
$tmp_1 \leftarrow z - y$		
return tmp_1		

SSA Construction – Local Value Numbering

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$x \leftarrow 5 + 3$	$v_1 \leftarrow \text{add } 5, 3$	$x \rightarrow v_3$!
$y \leftarrow x + 1$	$v_2 \leftarrow \text{add } v_1, 1$	$y \rightarrow v_2$
$\rightarrow x \leftarrow 12$	$v_3 \leftarrow \text{const } 12$	
$z \leftarrow x + 1$		
$tmp_1 \leftarrow z - y$		
return tmp_1		

SSA Construction – Local Value Numbering

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$x \leftarrow 12$	$v_3 \leftarrow \text{const } 12$	$z \rightarrow v_4$
$\rightarrow z \leftarrow x + 1$	$v_4 \leftarrow \text{add } v_3, 1$	
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$x \leftarrow 12$	$v_3 \leftarrow \text{const } 12$	$z \rightarrow v_4$
$z \leftarrow x + 1$	$v_4 \leftarrow \text{add } v_3, 1$	$tmp_1 \rightarrow v_5$
$\rightarrow tmp_1 \leftarrow z - y$	$v_5 \leftarrow \text{sub } v_4, v_2$	
return tmp_1		

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- ▶ Simple case: inside block – keep mapping of variable to value

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$x \leftarrow 5 + 3$	$v_1 \leftarrow \text{add } 5, 3$	$x \rightarrow v_3$
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$x \leftarrow 12$	$v_3 \leftarrow \text{const } 12$	$z \rightarrow v_4$
$z \leftarrow x + 1$	$v_4 \leftarrow \text{add } v_3, 1$	$tmp_1 \rightarrow v_5$
$tmp_1 \leftarrow z - y$	$v_5 \leftarrow \text{sub } v_4, v_2$	
$\rightarrow \text{return } tmp_1$	$\text{ret } v_5$	

SSA Construction – Local Value Numbering

- ▶ Simple case: inside block – keep mapping of variable to value

Code	SSA IR	Variable Mapping
$x \leftarrow 5 + 3$	$v_1 \leftarrow \text{add } 5, 3$	$x \rightarrow v_3$
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$x \leftarrow 12$	$v_3 \leftarrow \text{const } 12$	$z \rightarrow v_4$
$z \leftarrow x + 1$	$v_4 \leftarrow \text{add } v_3, 1$	$tmp_1 \rightarrow v_5$
$tmp_1 \leftarrow z - y$	$v_5 \leftarrow \text{sub } v_4, v_2$	
return tmp_1	ret v_5	

SSA Construction – Across Blocks

- ▶ SSA construction with control flow is non-trivial
- ▶ Key problem: find value for variable in predecessor
- ▶ Naive approach: Φ -nodes for all variables everywhere
 - ▶ Create empty Φ -nodes for variables, populate variable mapping
 - ▶ Fill blocks (as on last slide)
 - ▶ Fill Φ -nodes with last value of variable in predecessor

SSA Construction – Across Blocks

- ▶ SSA construction with control flow is non-trivial
- ▶ Key problem: find value for variable in predecessor
- ▶ Naive approach: Φ -nodes for all variables everywhere
 - ▶ Create empty Φ -nodes for variables, populate variable mapping
 - ▶ Fill blocks (as on last slide)
 - ▶ Fill Φ -nodes with last value of variable in predecessor
- ▶ Why is this a bad idea? \Rightarrow *don't do this!*
 - ▶ *Extremely inefficient, code size explosion, many dead Φ*

SSA Construction – Across Blocks (“simple”⁴)

- ▶ Key problem: find value in predecessor
- ▶ Idea: seal block once all direct predecessors are known
 - ▶ For acyclic constructs: trivial
 - ▶ For loops: seal header once loop block is generated
- ▶ Current block not sealed: add Φ -node, fill on sealing
- ▶ Single predecessor: recursively query that
- ▶ Multiple preds.: add Φ -node, fill now

SSA Construction – Example

```
func foo(v1)
```

```
int foo(int n) {  
    int res = 1;  
    while (n) {  
        res *= n * n;  
        n -= 1;  
    }  
    return res;  
}
```

SSA Construction – Example

```
func foo( $v_1$ )  
entry: sealed; varmap:  $n \rightarrow v_1$ 
```

```
int foo(int n) {  
    int res = 1;  
    while (n) {  
        res *= n * n;  
        n -= 1;  
    }  
    return res;  
}
```

SSA Construction – Example

```
func foo( $v_1$ )  
entry: sealed; varmap:  $n \rightarrow v_1, res \rightarrow v_2$   
 $v_2 \leftarrow 1$ 
```

```
int foo(int n) {  
    int res = 1;  
    while (n) {  
        res *= n * n;  
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    }  
    return res;  
}
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int foo(int n) {  
    int res = 1;  
    while (n) {  
        res *= n * n;  
        n -= 1;  
    }  
    return res;  
}
```

```
func foo( $v_1$ )  
entry: sealed; varmap:  $n \rightarrow v_1, res \rightarrow v_2$   
        $v_2 \leftarrow 1$   
header: NOT sealed; varmap:  $\emptyset$   
  
body:  NOT sealed; varmap:  $\emptyset$   
  
cont:  NOT sealed; varmap:  $\emptyset$ 
```

SSA Construction – Example

```
int foo(int n) {  
    int res = 1;  
    while (n) {  
        res *= n * n;  
        n -= 1;  
    }  
    return res;  
}
```

```
func foo(v1)  
entry: sealed; varmap: n→v1, res→v2  
v2 ← 1  
header: NOT sealed; varmap: ∅  
v3 ← equal ???, 0  
body: NOT sealed; varmap: ∅  
cont: NOT sealed; varmap: ∅
```

SSA Construction – Example

```
int foo(int n) {  
  int res = 1;  
  while (n) {  
    res *= n * n;  
    n -= 1;  
  }  
  return res;  
}
```

```
func foo( $v_1$ )  
entry: sealed; varmap:  $n \rightarrow v_1, res \rightarrow v_2$   
        $v_2 \leftarrow 1$   
header: NOT sealed; varmap:  $n \rightarrow \phi_1$   
         $\phi_1 \leftarrow \phi$  incomplete, for n  
         $v_3 \leftarrow \text{equal } \phi_1, 0$   
body:  NOT sealed; varmap:  $\emptyset$   
  
cont:  NOT sealed; varmap:  $\emptyset$ 
```

SSA Construction – Example

```
int foo(int n) {  
    int res = 1;  
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         $v_3 \leftarrow \text{equal } \phi_1, 0$   
        br  $v_3$ , cont, body  
body:  NOT sealed; varmap:  $\emptyset$   
  
cont:  NOT sealed; varmap:  $\emptyset$ 
```

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```
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    int res = 1;  
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        $v_2 \leftarrow 1$   
header: NOT sealed; varmap:  $n \rightarrow \phi_1$   
         $\phi_1 \leftarrow \phi$  incomplete, for n  
         $v_3 \leftarrow \text{equal } \phi_1, 0$   
        br  $v_3$ , cont, body  
body:  sealed; varmap:  $\emptyset$   
        $v_4 \leftarrow \text{mul } ???, ???$   
  
cont:  NOT sealed; varmap:  $\emptyset$ 
```

SSA Construction – Example

```
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    int res = 1;  
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        br  $v_3$ , cont, body  
body:  sealed; varmap:  $n \rightarrow \phi_1$   
         $v_4 \leftarrow \text{mul } \phi_1, \phi_1$   
  
cont:  NOT sealed; varmap:  $\emptyset$ 
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SSA Construction – Example

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int foo(int n) {  
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        n -= 1;  
    }  
    return res;  
}
```

```
func foo(v1)  
entry: sealed; varmap: n → v1, res → v2  
v2 ← 1  
header: NOT sealed; varmap: n → φ1  
φ1 ← φ incomplete, for n  
v3 ← equal φ1, 0  
br v3, cont, body  
body: sealed; varmap: n → φ1  
v4 ← mul φ1, φ1  
v5 ← mul ???, v4  
cont: NOT sealed; varmap: ∅
```

SSA Construction – Example

```
int foo(int n) {  
    int res = 1;  
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     $\phi_1 \leftarrow \phi$  incomplete, for n  
     $\phi_2 \leftarrow \phi$  incomplete, for res  
     $v_3 \leftarrow \text{equal } \phi_1, 0$   
    br  $v_3$ , cont, body  
body: sealed; varmap:  $n \rightarrow \phi_1, res \rightarrow v_5$   
     $v_4 \leftarrow \text{mul } \phi_1, \phi_1$   
     $v_5 \leftarrow \text{mul } \phi_2, v_4$   
cont: NOT sealed; varmap:  $\emptyset$ 
```

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         $v_3 \leftarrow \text{equal } \phi_1, 0$   
        br  $v_3$ , cont, body  
body:  sealed; varmap:  $n \rightarrow v_6, res \rightarrow v_5$   
        $v_4 \leftarrow \text{mul } \phi_1, \phi_1$   
        $v_5 \leftarrow \text{mul } \phi_2, v_4$   
        $v_6 \leftarrow \text{sub } \phi_1, 1$   
cont:  NOT sealed; varmap:  $\emptyset$ 
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    br  $v_3$ , cont, body  
body: sealed; varmap:  $n \rightarrow v_6, res \rightarrow v_5$   
     $v_4 \leftarrow \text{mul } \phi_1, \phi_1$   
     $v_5 \leftarrow \text{mul } \phi_2, v_4$   
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    br header  
cont: NOT sealed; varmap:  $\emptyset$ 
```

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     $v_3 \leftarrow \text{equal } \phi_1, 0$   
    br  $v_3$ , cont, body  
  body: sealed; varmap:  $n \rightarrow v_6, res \rightarrow v_5$   
     $v_4 \leftarrow \text{mul } \phi_1, \phi_1$   
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    br header  
  cont: NOT sealed; varmap:  $\emptyset$ 
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           $\phi_1 \leftarrow \phi(\text{entry: } v_1, \text{body: } v_6)$   
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           $v_3 \leftarrow \text{equal } \phi_1, 0$   
          br  $v_3, \text{cont}, \text{body}$   
  body:  sealed; varmap:  $n \rightarrow v_6, res \rightarrow v_5$   
          $v_4 \leftarrow \text{mul } \phi_1, \phi_1$   
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         br header  
  cont:  sealed; varmap:  $\emptyset$ 
```


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```
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  entry: sealed; varmap:  $n \rightarrow v_1, res \rightarrow v_2$   
          $v_2 \leftarrow 1$   
  header: sealed; varmap:  $n \rightarrow \phi_1, res \rightarrow \phi_2$   
           $\phi_1 \leftarrow \phi(\text{entry: } v_1, \text{body: } v_6)$   
           $\phi_2 \leftarrow \phi(\text{entry: } v_2, \text{body: } v_5)$   
           $v_3 \leftarrow \text{equal } \phi_1, 0$   
          br  $v_3$ , cont, body  
  body:  sealed; varmap:  $n \rightarrow v_6, res \rightarrow v_5$   
          $v_4 \leftarrow \text{mul } \phi_1, \phi_1$   
          $v_5 \leftarrow \text{mul } \phi_2, v_4$   
          $v_6 \leftarrow \text{sub } \phi_1, 1$   
         br header  
  cont:  sealed; varmap:  $res \rightarrow \phi_2$   
         ret  $\phi_2$ 
```

SSA Construction – Pruned/Minimal Form


- ▶ Resulting SSA is *pruned* – all ϕ are used
- ▶ But not *minimal* – ϕ nodes might have single, unique value


⁵M Braun et al. "Simple and efficient construction of static single assignment form". In: *CC*. 2013, pp. 102–122. .

⁶R Cytron et al. "Efficiently computing static single assignment form and the control dependence graph". In: *TOPLAS* 13.4 (1991), pp. 451–490. .

SSA Construction – Pruned/Minimal Form

- ▶ Resulting SSA is *pruned* – all ϕ are used
- ▶ But not *minimal* – ϕ nodes might have single, unique value
- ▶ When filling ϕ , check that multiple real values exist
 - ▶ Otherwise: replace ϕ with the single value
 - ▶ On replacement, update all ϕ using this value, they might be trivial now, too
- ▶ Sufficient?


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- ▶ When filling ϕ , check that multiple real values exist
 - ▶ Otherwise: replace ϕ with the single value
 - ▶ On replacement, update all ϕ using this value, they might be trivial now, too
- ▶ Sufficient? Not for irreducible CFG
 - ▶ Needs more complex algorithms⁵ or different construction method⁶

AD IN2053 “Program Optimization” covers this more formally

⁵M Braun et al. “Simple and efficient construction of static single assignment form”. In: *CC*. 2013, pp. 102–122. .

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SSA: Implementation

- ▶ Value is often just a pointer to instruction
- ▶ ϕ nodes placed at beginning of block
 - ▶ They execute “concurrently” and on the edges, after all

SSA: Implementation

- ▶ Value is often just a pointer to instruction
- ▶ ϕ nodes placed at beginning of block
 - ▶ They execute “concurrently” and on the edges, after all
- ▶ Variable number of operands required for ϕ nodes
- ▶ Storage format for instructions and basic blocks
 - ▶ Consecutive in memory: hard to modify/traverse
 - ▶ Array of pointers: $\mathcal{O}(n)$ for a single insertion...
 - ▶ Linked List: easy to insert, but pointer overhead

Is SSA a graph IR?

Is SSA a graph IR?

Only if instructions have no side effects,
consider `load`, `store`, `call`, ...

These *can* be solved using explicit dependencies as SSA values, e.g. for memory

Intermediate Representations – Summary

- ▶ An IR is an internal representation of a program
- ▶ Main goal: simplify analyses and transformations
- ▶ IRs typically based on graphs or linear instructions
- ▶ Graph IRs: AST, Control Flow Graph, Relational Algebra
- ▶ Linear IRs: stack machines, register machines, SSA
- ▶ Single Static Assignment makes data flow explicit
- ▶ SSA is extremely popular, although non-trivial to construct

Intermediate Representations – Questions

- ▶ Who designs an IR? What are design criteria?
- ▶ Why is an AST not suited for program optimization?
- ▶ How to convert an AST to another IR?
- ▶ What are the benefits/drawbacks of stack/register machines?
- ▶ What benefits does SSA offer over a normal register machine?
- ▶ How do ϕ -instructions differ from normal instructions?